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C17-1

**EXPERIMENTAL STUDY OF VIBRATION TRANSMISSIBILITY
USING CHARACTERIZATION OF COMPRESSOR MOUNTING
GROMMETS, DYNAMIC STIFFNESSES
PART-I, FREQUENCY RESPONSE TECHNIQUE DEVELOPMENT,
ANALYTICAL**

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ABSTRACT

The main sources that contribute to scroll compressor vibration at running frequency are rotational unbalance in the crank-rotor assembly and the reciprocating force due to the Oldham Ring. A rigid body model of the compressor vibrating on resilient grommets has been developed in this paper. The model is used to study the effect of different grommets for vibration suppression of the compressor as well as the reduction of vibration transmissibility to the base panel of a system (e.g. heat pump or condensing unit). This study enables us to select grommets appropriately to provide the desired vibration isolation for a system.

NOMENCLATURE

XG, YG, ZG	Coordinate at center of gravity, G, of Compressor
M	Compressor mass
K	Spring rate or stiffness
C	Damping coefficient
T	Time
f	Frequency, Hz
f_n	Natural frequency, Hz
f_{dr}	Driving or excitation frequency, Hz
ω	Angular Frequency = $2(\pi)f$
$x(t), y(t), z(t)$	Displacements or translations along XG, YG, ZG axis
$\dot{x}(t), \dot{y}(t), \dot{z}(t)$	Velocities along XG, YG, ZG axis
$\ddot{x}(t), \ddot{y}(t), \ddot{z}(t)$	Acceleration along XG, YG, ZG axis
$\xi = \frac{C}{C_c} = \frac{C}{2M\omega_n}$	Damping ratio
$r = \frac{\omega_{dr}}{\omega_n}$	Frequency ratio
TR	Vibration Transmissibility
$\eta = \frac{j\omega_{dr}C}{K} \cong 2\xi$	Loss Factor
E	Real part of the dynamic modulus of elasticity of the elastomeric materials.
G	Real part of the dynamic shear modulus of the elastomeric materials.
ν	Poisson's ratio and can be considered approximately.

θ_x	Angular displacement about XG-axis, Radian
ω_x	Angular frequency, Radian/second
θ_y	Angular displacement about YG-axis, Radian
ω_y	Angular frequency, Radian/second
θ_z	Angular displacement about ZG-axis, Radian
ω_z	Angular frequency, Radian/second
F_M	Maximum amplitude of the driving or excitation force
F_T	Maximum amplitude of the Transmitted force through mounting grommet
M_x, M_y, M_z	Moment about X, Y, Z axis respectively
L_G	The axial distance between compressor foot and center of gravity.
K_s and K_c	Dynamic shear and compression stiffness respectively

INTRODUCTION

Vibration can often lead to a number of undesirable effects. Structural or mechanical failure can often result from fatigue effects of sustained vibration. Electronic components may fail as well because of continuing vibration. Also, structural borne noise is usually produced as a consequence of maintained vibration. In designing a compressor, the desired vibrating response must be clearly stated. Different methods of measuring and describing acceptable levels of vibration have been proposed. The criteria should be defined in terms of displacement, velocity, or acceleration. Unwanted vibration can be reduced or even stopped by redesigning the source of vibration or even the compressor itself. It is also sometimes possible to design a vibration isolation system to isolate the compressor(s) from the air conditioning or refrigeration system(s) of interest. Using highly damped materials such as rubber grommet to change the stiffness and damping between the compressor and base panel of the system can achieve the latter. In this paper, transmissibility of the excitation loads that can be produced by a HVAC compressor has been measured and analyzed. The frequency response function technique has been used for analytical and experimental modal analysis.

FORCED VIBRATION OF A SINGLE DEGREE OF FREEDOM WITH DAMPING

The design of a mounting system is highly complex. A simple case is given as an illustration in Figures 1 & 2. The suspended compressor on its mounting is subjected to a vertical forced vibration which imposes a sinusoidal force at frequency ω_{dr} . A compressor with mass M may typically move parallel to the vertical axis Gz . It is fixed at its base by an elastic mounting system S whose stiffness along Gz is K .

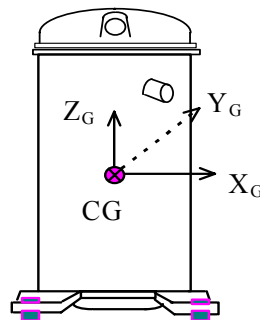


Fig. 1: Compressor on the Grommets

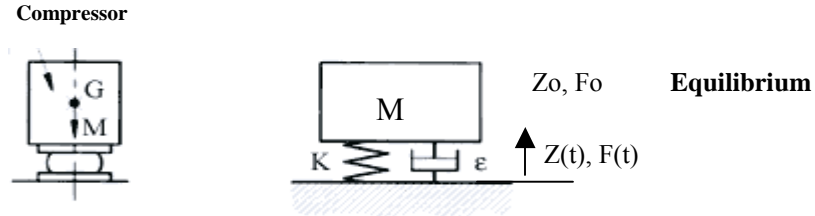


Figure-2: Mounting system

Mounting types: The mounting types that are discussed in this paper are: 1- *Rigidly Mounted Equipment*, in this case the whole disturbing force will be transmitted into the equipment. 2- *Resilient Mounted Equipment*: in this case at the start of the forced vibration at frequency ω_{dr} , a vibration is induced at the natural frequency ω_n . This is damped very quickly so that, within a very short time, only the vibration at the forcing frequency ω_{dr} (steady state motion) remains as a permanent vibration transmitting a sinusoidal force to the system.

1.2 Transmissibility of a Single Degree of Freedom System: Transmissibility is defined as a non-dimensional ratio of the response amplitude of a system in steady-state forced vibration to the excitation amplitude. The ratio may be one of forces, displacements, velocities or accelerations. The transmissibility of the grommet mounting is a function of the relationship between the driving or excitation frequency (input) and the system natural frequency, as well as damping values.

Notes: 1 - High frequency vibration (high ω) may therefore cause very high acceleration, even at low input amplitude. 2 - For natural rubber, the damping ratio is much smaller than one. Therefore damped natural frequency, ω_d , is very close to ω_n . Damped natural frequency is defined as $\omega_d = \omega_n \sqrt{1 - \xi^2}$.

3 - It must be noted that the addition of damping reduces the amount of protection in the isolation region of the transmissibility.

At steady state condition, the force transmitted can be defined as

Mounting	Exciting force	Force transmitted	Transmissibility
Rigid mounting system	$f(t) = F_M \sin \omega t$	$f(t) = F_T \sin \omega t$	$TR = 1$
Elastic mounting system	$f(t) = F_M \sin \omega t$	$f(t) = F'_T \sin \omega t$	$TR = \frac{F_T}{F_M} = \sqrt{\frac{1 + 4(\xi r)^2}{(1 - r^2)^2 + 4(\xi r)^2}}$ (1)

For low damping isolation materials (such as natural rubbers), damping ratio between 0.01 to 0.1, Equation (1) becomes

$$TR \cong \frac{1}{r^2 - 1} \quad (2)$$

1.1 Attenuation: For vibration suppression, mountings with low damping coefficient, ξ^2 is much smaller than one. The percentage attenuation is equal to 100 less the transmissibility coefficient. For a given excitation frequency ω_{dr} , the attenuation depends on the natural frequency of the mounting system.

$$1 - TR = \frac{r^2 - 2}{r^2 - 1} \quad (3)$$

3-DIMENSIONAL TRANSMISSIBILITY ANALYSIS USING FREQUENCY RESPONSE FUNCTION TECHNIQUE, SIX DEGREE OF FREEDOM SYSTEM

Theory: The compressor on four mounting feet is considered as a system with six-degree of freedom. The system with multi-degree of freedom is used to show how the frequency response functions of the system structure are related to the modal vectors of that structure. The frequency response function measurements are used as the basis for defining modal frequencies and damping values, modal vectors, modal mass, modal stiffness, and modal damping of the real life structures. Also, simulation or prediction of the structural response due to specified excitation-loading functions utilizing the above dynamic properties of the structure will be considered. To accomplish this task, an analytical model will be developed to represent the transfer function between any possible measurement locations on the structure. The frequency response function technique is also used for transmissibility of the excitation load from the compressor through the mounting to the base pan of the system. The analytical model representation of a six-degree of freedom system has been formulated by stating a system of six coupled non-homogeneous differential equations of motion as follows:

$$[M] \left\{ \ddot{u}(t) \right\} + [C] \left\{ \dot{u}(t) \right\} + [K] \left\{ u(t) \right\} = \{ f(t) \}$$

For low damping the system it can be written approximately as

$$[M] \left\{ \ddot{u}(t) \right\} + [K] \left\{ u(t) \right\} = \{ f(t) \} \quad (4)$$

Where:

- Excitation load matrix, $f(t) = [F_x(t), F_y(t), F_z(t), M_x(t), M_y(t), M_z(t)]$.
- System response matrix, $u(t) = [x(t), y(t), z(t), \theta_x(t), \theta_y(t), \theta_z(t)]$.
- Mass matrix, $[M] = \text{Diag} [m \quad m \quad m \quad I_x \quad I_y \quad I_z]$

$$\theta_x = \omega_x t, \quad \theta_y = \omega_y t, \quad \theta_z = \omega_z t \text{ Radian}$$

Taking Laplace transform of Equation (4), assuming all initial conditions are zero, yields

$$[s^2 [M] + [K]] \{ U(s) \} = \{ F(s) \} \quad (5)$$

$[s^2 [M] + [K]] = B(s)$ is the system impedance matrix or the system matrix. Therefore, Equation (5) becomes

$$[B(s)] \{ U(s) \} = F(s)$$

Pre-multiplying Equation (5) by $[B(s)]^{-1}$ yields

$$\{ U(s) \} = [B(s)]^{-1} \{ F(s) \}, \quad \{ H(s) \} = [B(s)]^{-1}$$

Where $\mathbf{H}(\mathbf{S})$ is the *transfer function matrix* of the system, therefore the equation of motion for the system and the transfer function become

$$\begin{aligned}\{U(s)\} &= [H(s)]\{F(s)\} \\ \frac{\{U(s)\}}{\{F(s)\}} &= [H(s)]\end{aligned}\quad (6)$$

Where the system transfer function matrix for this six-degree of freedom system is defined as

$$[H_{pq}(s)] = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & H_{16} \\ H_{21} & H_{22} & H_{23} & H_{24} & H_{25} & H_{26} \\ H_{31} & H_{32} & H_{33} & H_{34} & H_{35} & H_{36} \\ H_{41} & H_{42} & H_{43} & H_{44} & H_{45} & H_{46} \\ H_{51} & H_{52} & H_{53} & H_{54} & H_{55} & H_{56} \\ H_{61} & H_{62} & H_{63} & H_{64} & H_{65} & H_{66} \end{bmatrix} \quad (7) \quad p \text{ and } q=1,2,\dots,6$$

The determinant of $[B(s)]$ produces the characteristic equation of the system as

$$|B(s)| = D(s - \lambda_1)(s - \lambda_1^*)(s - \lambda_2)(s - \lambda_2^*)(s - \lambda_3)(s - \lambda_3^*)(s - \lambda_4)(s - \lambda_4^*)(s - \lambda_5)(s - \lambda_5^*)(s - \lambda_6)(s - \lambda_6^*) \quad (8)$$

Where D is a constant and λ and λ^* are complex conjugate roots of the characteristic equation of this system. They are also called poles of the system transfer function.

Using the partial fraction approach gives the H_{pq} for this six-degree of freedom system. For example H_{11} for the first pole or eigenvalue becomes:

$$H_{11} = \frac{X_1(s)}{F_1(s)} = \frac{C_1}{(s - \lambda_1)} + \frac{C_2}{(s - \lambda_1^*)} + \frac{C_3}{(s - \lambda_2)} + \frac{C_4}{(s - \lambda_2^*)} + \dots + \frac{C_{11}}{(s - \lambda_6)} + \frac{C_{12}}{(s - \lambda_6^*)}$$

$$C_1 = \frac{M_{11}^\circ \lambda_1^2 + K_{11}^\circ}{D(\lambda_1 - \lambda_1^*)(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_2^*) \dots (\lambda_1 - \lambda_6^*)(\lambda_1 - \lambda_6^*)} = A_{111}$$

$$C_2 = \frac{M_{11}^\circ \lambda_1^{*2} + K_{11}^\circ}{D(\lambda_1^* - \lambda_1)(\lambda_1^* - \lambda_2)(\lambda_1^* - \lambda_2^*) \dots (\lambda_1^* - \lambda_6)(\lambda_1^* - \lambda_6^*)} = A_{111}^*$$

•
•

$$C_{11} = \frac{M_{66}^\circ \lambda_1^2 + K_{66}^\circ}{D(\lambda_6 - \lambda_1)(\lambda_6 - \lambda_1^*)(\lambda_6 - \lambda_2)(\lambda_6 - \lambda_2^*) \dots (\lambda_6 - \lambda_6^*)} = A_{661}$$

$$C_{12} = \frac{M_{66}^\circ \lambda_1^{*2} + K_{66}^\circ}{D(\lambda_6 - \lambda_1)(\lambda_6 - \lambda_1^*)(\lambda_6 - \lambda_2)(\lambda_6 - \lambda_2^*) \dots (\lambda_6^* - \lambda_6)} = A_{661}^*$$

Where M_{pq}° & K_{pq}° are the elements of $[B(s)]^{-1}$ that construct H_{pq} , and A_{pq} , and are called residue terms.

The dynamic stiffness matrix for each mounting foot of the compressor is presented by $[k_i]$.

$$[k_i] = \begin{bmatrix} k_s & 0 & 0 & 0 & -k_s L_G & -Rk_s \sin(\alpha_i) \\ 0 & k_s & 0 & k_s L_G & 0 & Rk_s \cos(\alpha_i) \\ 0 & 0 & k_c & Rk_c \sin(\alpha_i) & -Rk_c \cos(\alpha_i) & 0 \\ 0 & k_s L_G & Rk_c \sin(\alpha_i) & k_s L_G^2 + R^2 k_c \sin^2(\alpha_i) & -R^2 k_c \sin(\alpha_i) \cos(\alpha_i) & RL_G k_s \cos(\alpha_i) \\ -k_s L_G & 0 & -Rk_c \cos(\alpha_i) & -R^2 k_c \sin(\alpha_i) \cos(\alpha_i) & k_s L_G^2 + R^2 k_c \cos^2(\alpha_i) & RL_G k_s \sin(\alpha_i) \\ -Rk_s \sin(\alpha_i) & Rk_s \cos(\alpha_i) & 0 & RL_G k_s \cos(\alpha_i) & RL_G k_s \sin(\alpha_i) & R^2 k_s \end{bmatrix} \quad (9)$$

Therefore, the system mounting stiffness matrix for four compressor feet can be written as

$$[K] = \sum_{i=1}^4 [k_i] \quad (10)$$

Where $\alpha_i = \alpha_1 + (i-1)\frac{\pi}{2}$ and α_1 is the angle between the location of foot-1 of the compressor and X-axis.

Note: Similar relations can be developed for the mounting damping and using it into the six-degree of freedom equation of motion for a damped system as follows:

$$[c_i] = \begin{bmatrix} c_s & 0 & 0 & 0 & -c_s L_G & -Rc_s \sin(\alpha_i) \\ 0 & c_s & 0 & c_s L_G & 0 & Rc_s \cos(\alpha_i) \\ 0 & 0 & c_c & Rc_c \sin(\alpha_i) & -Rc_c \cos(\alpha_i) & 0 \\ 0 & c_s L_G & Rc_c \sin(\alpha_i) & c_s L_G^2 + R^2 c_c \sin^2(\alpha_i) & -R^2 c_c \sin(\alpha_i) \cos(\alpha_i) & RL_G c_s \cos(\alpha_i) \\ -c_s L_G & 0 & -Rc_c \cos(\alpha_i) & -R^2 c_c \sin(\alpha_i) \cos(\alpha_i) & c_s L_G^2 + R^2 c_c \cos^2(\alpha_i) & RL_G c_s \sin(\alpha_i) \\ -Rc_s \sin(\alpha_i) & Rc_s \cos(\alpha_i) & 0 & RL_G c_s \cos(\alpha_i) & RL_G c_s \sin(\alpha_i) & R^2 c_s \end{bmatrix} \quad (11)$$

$$[C] = \sum_{i=1}^4 [c_i] \quad (12)$$

$$H(s) = \frac{\begin{bmatrix} A_{111} & A_{121} \\ A_{211} & A_{221} \end{bmatrix}}{(s - \lambda_1)} + \frac{\begin{bmatrix} A_{111}^* & A_{121}^* \\ A_{211}^* & A_{221}^* \end{bmatrix}}{(s - \lambda_1^*)} + \frac{\begin{bmatrix} A_{112} & A_{122} \\ A_{212} & A_{222} \end{bmatrix}}{(s - \lambda_2)} + \frac{\begin{bmatrix} A_{112}^* & A_{122}^* \\ A_{212}^* & A_{222}^* \end{bmatrix}}{(s - \lambda_2^*)} + \dots + \frac{\begin{bmatrix} A_{115} & A_{125} \\ A_{215} & A_{225} \end{bmatrix}}{(s - \lambda_5)} + \frac{\begin{bmatrix} A_{115}^* & A_{125}^* \\ A_{215}^* & A_{225}^* \end{bmatrix}}{(s - \lambda_5^*)} + \frac{\begin{bmatrix} A_{116} & A_{126} \\ A_{216} & A_{226} \end{bmatrix}}{(s - \lambda_6)} + \frac{\begin{bmatrix} A_{116}^* & A_{126}^* \\ A_{216}^* & A_{226}^* \end{bmatrix}}{(s - \lambda_6^*)} \quad (13)$$

Note: The system poles for low damping values move close to the imaginary axis. Therefore, for low damping conditions, the Laplace complex operator that is defined as $s = -\xi\omega \pm j\omega\sqrt{1-\xi^2}$ can be approximated as $s \approx \pm j\omega$. Therefore the equation of motion, Equation (4), can be a good approximation

Transmitted force through the grommets to the base pan of the system is approximately $F_T = [K]\{U_{max}\}$, neglecting damping.

Where:

$$\begin{aligned}\{U_{max}(\omega)\} &= \{F_{max}(\omega)\}[H(\omega)] \\ \{U_{max}\} &= [X_{max}, Y_{max}, Z_{max}, \theta_{max-x}, \theta_{max-y}, \theta_{max-z}]^T \\ \{F_{max}\} &= [F_{max-x}, F_{max-y}, F_{max-z}, M_{max-x}, M_{max-y}, M_{max-z}]^T\end{aligned}$$

Therefore,
$$TR = \frac{F_T}{F_{max}} \quad (14)$$

The following procedure can be used for modal analysis:

$$\begin{bmatrix} A_{1r} & A_{1z} & \bullet & \bullet & \bullet & A_{1\theta} \\ A_{2r} & A_{2z} & \bullet & \bullet & \bullet & A_{2\theta} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ A_{6r} & A_{6z} & \bullet & \bullet & \bullet & A_{6\theta} \end{bmatrix} = Q \begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \varphi_5 \\ \varphi_6 \end{Bmatrix}_r \begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \varphi_5 \\ \varphi_6 \end{Bmatrix}_r^T = L \begin{bmatrix} (M_{22}\varphi_r^2 + K_{22}) & -(M_{12}\varphi_r^2 + K_{12}) & \bullet & \bullet & \bullet & -(M_{16}\varphi_r^2 + K_{16}) \\ -(M_{21}\varphi_r^2 + K_{21}) & (M_{22}\varphi_r^2 + K_{22}) & \bullet & \bullet & \bullet & -(M_{26}\varphi_r^2 + K_{26}) \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ -(M_{61}\varphi_r^2 + K_{61}) & -(M_{62}\varphi_r^2 + K_{62}) & \bullet & \bullet & \bullet & (M_{66}\varphi_r^2 + K_{66}) \end{bmatrix} \quad (15)$$

$r = \text{mode \#} = 1, 2, \dots, 6$

$$\begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \varphi_5 \\ \varphi_6 \end{Bmatrix}_r \begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \varphi_5 \\ \varphi_6 \end{Bmatrix}_r^T = \begin{bmatrix} \varphi_1\varphi_1 & \varphi_1\varphi_2 & \bullet & \bullet & \bullet & \varphi_1\varphi_6 \\ \varphi_1\varphi_2 & \varphi_2\varphi_2 & \bullet & \bullet & \bullet & \varphi_1\varphi_6 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \varphi_6\varphi_1 & \varphi_6\varphi_2 & & & & \varphi_6\varphi_6 \end{bmatrix}_r \quad (16)$$

Q and L are proportionally constant values, therefore they can be considered unity because the amplitudes of mode shapes are arbitrary relative values. The 6x1 column modal vector $\{\varphi_r\}_r$ can be calculated using only one row or one column of the right side matrix of Equation (16).

Notations: 1- The response of equipment on anti-vibration mounts to a random vibration environment often requires complex analysis. However, for the simple case of equipment mounted so that the response at resonance is uncoupled, i.e., translation without significant rotation, the response to a random vibration input will be vibration at almost constant frequency (i.e. the resonance frequency) with randomly varying amplitude. 2- For an input ASD of S (g^2 / Hz) that is constant, or nearly so, at the resonance frequency f_n (Hz), and if the magnification factor of mounts at resonance is Q, then the peak acceleration and displacement response with $r=3$ (statistical approach 3 sigma) will be, approximately:

$$\begin{aligned}
 A &\approx 3\sqrt{\frac{\pi S f_n Q}{2}}, gs \\
 Q &\approx \frac{1}{Loss - Factor} \\
 X &\approx \pm \frac{A}{(2\pi^2 f_n^2)9.81}, m
 \end{aligned}
 \tag{17}$$

Where “A” and “X” are acceleration-peak and displacement-peak

3- Elastomer parts typically respond sinusoidally when subjected to a random vibration input of 2σ . It is therefore advisable to use $r = 2$ (2 sigma) when determining resonant frequencies rather than $r = 3$ (3 sigma) which gives peak responses. 4- These equations can also be used to estimate the sway space requirement and displacement capability required of the mounts. The output amplitude at the resonance stud is the product of “X” and “Q”.

APPLICATIONS

The frequency response technique that has been developed in this paper (Part-I) can be used for the following applications:

- 1- Experimental and analytical vibration transmissibility measurements.
- 2- Experimental modal analysis of multi-degree of freedom systems with damping, both for proportional and non-proportional damping cases.
- 3- Also for experimental dynamic stiffness measurements of grommets.

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APPENDIX-A
(INFORMATIVE)

Complex Stiffness, K^* is defined as

$$K^* = K \left(1 + \frac{j\omega_{dr}C}{K} \right) = K(1 + j\eta), \text{ N/m}$$

Thus, the complex compression stiffness (spring rate) and complex shear stiffness can be defined as

$$K_c^* = K_c(1 + j\eta), \text{ N/m}, \quad K_s^* = K_s(1 + j\eta), \text{ N/m}, \quad K_t^* = K_t(1 + j\eta), \text{ N/m}$$

Torsional complex modulus of a viscoelastic hollow cylinder can be defined as

$$\begin{aligned} K_s &= \frac{(G)(\pi)(D_o^2 - D_i^2)}{4 * h} & \text{N/m} \\ K_c &= \frac{(E_c)(\pi)(D_o^2 - D_i^2)}{4 * h} & \text{N/m} \\ K_{torsion} &= \frac{(\pi)(G)(D_o^4 - D_i^4)}{32 * L} & \text{N-m/rad} \end{aligned}$$

Note: $\frac{(\pi)(D_o^2 - D_i^2)}{4}$ gives the loading area of the mounting grommet at the compressor feet.